

Econometric Model: Canada

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1. Model explanation

The aim of the model is to relate the Canadian short-term interest rate with inflation and a measure of real activity. In order to do this we used the Taylor rule.

$$i_t = \alpha + B_{\Pi}(\Pi_t - \Pi^*) + B_y Y_t$$

where α is the equilibrium interest rate (intercept).

In the R code, the Taylor regression is performed in this way:

```
taylor_reg_monthly <- lm((IntRates_monthly) ~ ((CPI_monthly) + (GDPgap_monthly)))
```

We took into account the period that spans from 2000 to 2022, so that we have a consistent and updated amount of data.

After the monthly Taylor regression, we produced some variations of the model taking into account different periods (i.e. annual, without covid) or adding other explanatory variables (i.e. unemployment, expected interest rates).

1. Descriptive analysis of interest rates

➤ Interest rate:

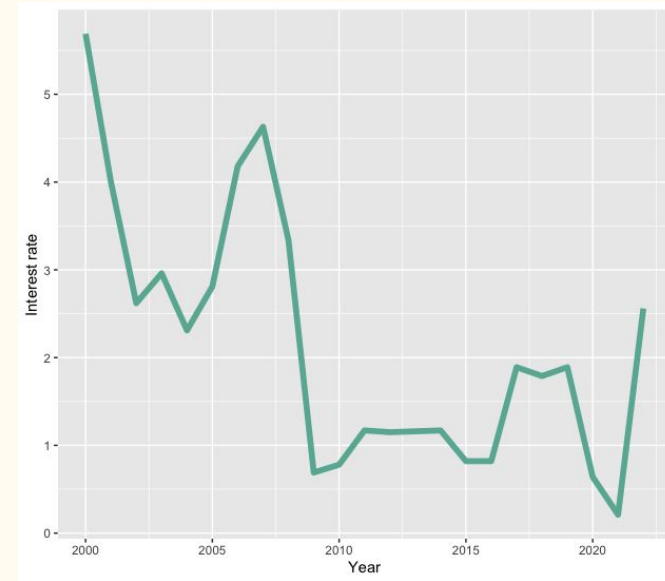
It indicates the amount paid for borrowing money, expressed as a percentage of the entire loan.

The graph shows how annual short-term interest rates fluctuate through our considered time span:

The interest rate was steadily decreasing until 2004, then it increased, peaking in 2007 at around 4.6%. Due to the Great Financial Crisis, the interest rate sharply declined between 2008 and 2009 reaching a value below 1%. Despite a minor recovery, in 2014, oil prices dropped, causing a recession in Canada's economy and a decrease in the interest rate. The years 2017- 2019 were characterized by economic growth but also low inflation and the interest rate was rather stable below 2%. From 2020 the interest rate was kept low for two years to contract the effect of the Covid pandemic on the economy. The expansionary monetary and fiscal policy implemented by Canadian government to ameliorate negative effects of the pandemic led to higher inflation and a sharp increase in the interest rate in 2022.

➤ Expected interest rate:

It's a rate that is forward-looking and offers information on potential investment opportunities and market conditions.



1. Descriptive analysis of the independent variables

➤ **Real activity: GDP gap**

The output gap refers to the discrepancy between the economy's actual output and the level of production that can be attained using the labor, capital, and technology already in place without consistently pushing inflation upward. The data is analyzed following the Integrated Framework measure, which takes into consideration long-term structural changes in the economy and calculates potential output as the product of trend labour input and trend labour productivity.

➤ **Inflation: Consumer Price Index**

Because it includes commodities that Canadians commonly purchase (e.g. food, housing, transportation, furniture, clothing, and recreational activities), CPI is the most pertinent indicator of the cost of living. The Bank of Canada targets an inflation-control goal range of 1 to 3%, with a 2% middle. The overall CPI increase over the previous year is used to calculate the inflation target.

➤ **Unemployment rate**

The unemployment rate is measured in numbers of unemployed people (people of working age who are without work, are available for work, and are actively looking for work) as a percentage of the labour force.

All the data is given as a percentage.

2. First model: using monthly data

```
Call:
lm(formula = (IntRates_monthly) ~ ((CPI_monthly) + (GDPgap_monthly)))
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-1.4480 -0.8043 -0.2991  0.4640  4.0218
```

Coefficients:

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    7.02436    0.60224  11.664 < 2e-16 ***
CPI_monthly   -0.03899    0.00513  -7.601 4.73e-13 ***
GDPgap_monthly 0.30810    0.03511   8.775 < 2e-16 ***
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 1.083 on 273 degrees of freedom
Multiple R-squared:  0.4932,    Adjusted R-squared:  0.4895
F-statistic: 132.8 on 2 and 273 DF,  p-value: < 2.2e-16
```

If we analyze the results we got from the first Taylor regression, we can affirm that the levels of significance are acceptable. However, this model has $R^2 = 0.4932$, which is poor.

Regarding the model parameters, we have that a unit increase in the monthly CPI is associated with a decrease of 0.03899 units in the interest rate, whereas a unit increase in the monthly GDP gap is associated with an increase of 0.30810 units in the interest rate.

We decided to apply the RESET test for linearity, the Breusch-Pagan test for homoscedasticity and Breusch-Godfrey test for serial correlation for this model and the following ones.

We can see that the p-values resulting from the 3 tests are smaller than 0.05, therefore this means we are failing all the tests.

RESET test

```
data: taylor_reg_monthly
RESET = 127.98, df1 = 2, df2 = 271, p-value < 2.2e-16
```

studentized Breusch-Pagan test

```
data: taylor_reg_monthly
BP = 7.7995, df = 2, p-value = 0.02025
```

Breusch-Godfrey test for serial correlation of order up to 1

```
data: taylor_reg_monthly
LM test = 255.22, df = 1, p-value < 2.2e-16
```

3a. Alternative model specifications: using annual data

```
Call:
lm(formula = (IntRates_annual) ~ ((CPI_annual) + (GDPgap_annual)))
```

Residuals:

Min	1Q	Median	3Q	Max
-1.3648	-0.5604	0.1819	0.5246	1.4574

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	8.33119	1.32606	6.283	3.91e-06 ***
CPI_annual	-0.04912	0.01110	-4.424	0.000261 ***
GDPgap_annual	0.71884	0.13141	5.470	2.35e-05 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7621 on 20 degrees of freedom
Multiple R-squared: 0.7524, Adjusted R-squared: 0.7276
F-statistic: 30.38 on 2 and 20 DF, p-value: 8.673e-07

We decided to use annual data to reduce the effects of short-term fluctuations given by the monthly dataset.

We can see that the levels of significance are still adequate and now the value of R^2 is good.

Regarding the model parameters, we have that a unit increase in the annual CPI is associated with a decrease of 0.04912 units in the interest rate, whereas a unit increase in the annual GDP gap is associated with an increase of 0.71884 units in the interest rate.

When checking the tests, we can affirm that the model only fails homoscedasticity.

Therefore, we can state that using annual data is better than using monthly data.

RESET test

data: taylor_reg_annual
RESET = 3.4554, df1 = 2, df2 = 18, p-value = 0.0537

studentized Breusch-Pagan test

data: taylor_reg_annual
BP = 8.1922, df = 2, p-value = 0.01664

Breusch-Godfrey test for serial correlation of order up to 1

data: taylor_reg_annual
LM test = 0.19198, df = 1, p-value = 0.6613

3b. Adding annual unemployment rate

```
Call:
lm(formula = (IntrRates_annual) ~ ((CPI_annual) + (GDPgap_annual) +
  (Unemployment)))
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-0.8730 -0.3051 -0.1015  0.4094  0.9654
```

Coefficients:

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  13.467120   1.667927   8.074 1.46e-07 ***
CPI_annual   -0.058980   0.008878  -6.644 2.35e-06 ***
GDPgap_annual 0.532770   0.111498   4.778 0.000131 ***
Unemployment -0.577602   0.148769  -3.883 0.001002 **
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 0.5838 on 19 degrees of freedom

Multiple R-squared: 0.8619, Adjusted R-squared: 0.8401

F-statistic: 39.53 on 3 and 19 DF, p-value: 2.292e-08

Since in the previous model we failed homoscedasticity, we concluded that it wasn't well defined. Thus, we considered it appropriate to add another explanatory variable: the unemployment rate. In fact, as shown by the Phillips Curve, there is an inverse relationship between unemployment rate and inflation.

When adding unemployment to our annual model, we can see that R^2 is a bit larger. However, we have only 2 stars for unemployment so it is significant at 1% level.

Regarding the model parameters, we have that a unit increase in the annual CPI is associated with a decrease of 0.058980 units in the interest rate, whereas a unit increase in the annual GDP gap is associated with an increase of 0.532770 units in the interest rate and a unit increase in the annual unemployment is associated with a decrease of 0.577602 units in the interest rate.

When checking the tests, we can say that the model is not linear, but passes homoscedasticity and serial correlation.

RESET test

```
data: taylor_reg_unemployment
RESET = 4.4356, df1 = 2, df2 = 17, p-value = 0.02818
```

studentized Breusch-Pagan test

```
data: taylor_reg_unemployment
BP = 4.5798, df = 3, p-value = 0.2053
```

Breusch-Godfrey test for serial correlation of order up to 1

```
data: taylor_reg_unemployment
LM test = 0.15941, df = 1, p-value = 0.6897
```

3c. Using the annual expected interest rate

```
Call:
lm(formula = (ExpIntRate_annual) ~ ((CPI_annual) + (GDPgap_annual)))
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-1.0935 -0.4782 -0.2624  0.1303  2.0497
```

Coefficients:

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  12.39011    1.42380   8.702  3.1e-08 ***
CPI_annual   -0.08943    0.01192  -7.502  3.1e-07 ***
GDPgap_annual 0.55440    0.14109   3.929  0.00083 ***
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 0.8182 on 20 degrees of freedom
```

```
Multiple R-squared:  0.8111,    Adjusted R-squared:  0.7922
```

```
F-statistic: 42.93 on 2 and 20 DF,  p-value: 5.789e-08
```

Coming back to our annual model (3a), we decided to use the expected interest rate to account for unforeseen circumstances.

We notice that the model provides a good R^2 (even though it's lower than the one we got by adding unemployment) and acceptable levels of significance.

Regarding the model parameters, we have that a unit increase in the annual CPI is associated with a decrease of 0.08943 units in the expected interest rate, whereas a unit increase in the annual GDP gap is associated with an increase of 0.55440 units in the expected interest rate.

When checking the tests, we can say that the model doesn't pass serial correlation, but passes linearity and homoscedasticity.

RESET test

```
data: taylor_reg_expIR
RESET = 2.5443, df1 = 2, df2 = 18, p-value = 0.1064
```

studentized Breusch-Pagan test

```
data: taylor_reg_expIR
BP = 1.7195, df = 2, p-value = 0.4233
```

Breusch-Godfrey test for serial correlation of order up to 1

```
data: taylor_reg_expIR
LM test = 8.4889, df = 1, p-value = 0.003573
```

3d. Removing the COVID time period

```
Call:
lm(formula = (IntRates_nocovid) ~ ((CPI_nocovid) + (GDPgap_nocovid)))
```

Residuals:

Min	1Q	Median	3Q	Max
-0.9943	-0.5040	0.1698	0.4603	0.9070

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	8.01210	1.47613	5.428	4.52e-05	***
CPI_nocovid	-0.04582	0.01291	-3.548	0.00247	**
GDPgap_nocovid	0.74923	0.11884	6.304	7.92e-06	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6183 on 17 degrees of freedom
Multiple R-squared: 0.8386, Adjusted R-squared: 0.8197
F-statistic: 44.18 on 2 and 17 DF, p-value: 1.845e-07

Our data encompassed the years of COVID outbreak - an infrequent event which could potentially cause temporary changes in the relationship between economic variables leading to parameter instability. This is why we decided to remove the COVID time period.

We can see that the R^2 is good, and the model gives good, even though not perfect, levels of significance: we have only 2 stars for the CPI, so it is significant at 1% level.

As for the discussion of the tests, it can be found in the next slide.

4. Diagnostic checks on our preferred model

RESET test

```
data: taylor_reg_nocovid  
RESET = 3.6811, df1 = 2, df2 = 15, p-value = 0.05004
```

studentized Breusch-Pagan test

```
data: taylor_reg_nocovid  
BP = 4.2605, df = 2, p-value = 0.1188
```

Breusch-Godfrey test for serial correlation of order up to 1

```
data: taylor_reg_nocovid  
LM test = 1.6251, df = 1, p-value = 0.2024
```

When checking the tests of our preferred model, we can see that it passes all the tests, satisfying the OLS assumptions.

5. Conclusion

Regarding our preferred model's parameters, we have that a unit increase in the annual CPI is associated with a decrease of 0.04582 units in the interest rate, whereas a unit increase in the annual GDP gap is associated with an increase of 0.74923 units in the interest rate.

In conclusion, as we previously said, our preferred model is the one where we removed the COVID time period because its associated lockdowns and economic disruptions represent an exogenous shock to the economy that is not captured by the previous Taylor rule models.

6. Bibliography

Interest Rate data:

<https://data.oecd.org/interest/short-term-interest-rates.htm>

CPI data:

<https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1810000501&pickMembers%5B0%5D=1.2&cubeTimeFrame.startYear=2000&cubeTimeFrame.endYear=2022&referencePeriods=20000101%2C20220101>

GDP gap data:

<https://www.bankofcanada.ca/rates/indicators/capacity-and-inflation-pressures/product-market-definitions/>
https://data.nasdaq.com/data/ODA/CAN_NGAP_NPGDP-canada-output-gap-of-potential-gdp

Unemployment data:

<https://data.worldbank.org/indicator/SL.UEM.TOTL.ZS?end=2021&locations=CA&start=2014>

Expected Interest Rate data:

<https://data.oecd.org/interest/short-term-interest-rates-forecast.htm#indicator-chart>